

线性关系与线性方程组

一、线性相关性与齐次线性方程组

结论1 设 $\alpha_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{pmatrix}, \alpha_2 = \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{n2} \end{pmatrix}, \dots, \alpha_m = \begin{pmatrix} a_{1m} \\ a_{2m} \\ \vdots \\ a_{nm} \end{pmatrix}, A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix}$ 则下列条件等价

(1) $\alpha_1, \alpha_2, \dots, \alpha_m$ 线性相关(无关).

(2) 存在不全为零的数 $k_1, k_2, \dots, k_m$ 使得 $k_1\alpha_1 + k_2\alpha_2 + \dots + k_m\alpha_m = 0$

(若 $k_1\alpha_1 + k_2\alpha_2 + \dots + k_m\alpha_m = 0$, 则 $k_1 = k_2 = \dots = k_m = 0$).

(3) 齐次方程组 $\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1m}x_m = 0, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2m}x_m = 0, \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nm}x_m = 0 \end{cases}$ 有非零解(只有零解).

(4) 系数矩阵 A 的秩 $< m (=m)$.

